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Probing the anisotropic velocity of light in a gravitational field: another test of general relativity

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Abstract

A corollary of general relativity that the velocity of light is anisotropic in a gravitational field has received little attention so far. It is shown that this anisotropy can be directly probed by an experiment initially proposed for testing the one-way velocity of light. The proposed experiment constitutes another test of general relativity since the anisotropy in the propagation of light in a gravitational field is not a new effect but follows from general relativity. This experiment is also an indirect test of the local constancy of the velocity of light which is a key concept of the standard interpretation of general relativity.

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Although there exist two clear indications that the propagation of light in the vicinity of massive bodies is anisotropic - the bending of light and the zero velocity of light which tries to leave a gravitationally collapsing body (a black hole) - the anisotropy in the velocity of light in a gravitational field has been only barely mentioned [1], [2]. If, however, the average velocity of light toward and away from a gravitational center between two points of different gravitational potential is calculated, it becomes clear that the two velocities are not the same. In general, the anisotropy in the propagation of light in a non-inertial frame of reference (accelerating or supported in a gravitational field) can be demonstrated by using a version of the classical thought experiment [3] involving two Einstein elevators - one accelerating with an uniform acceleration $\mathbf{a}$ and one at rest in a gravitational field of strength $\mathbf{g} = -\mathbf{a}$.

Consider an elevator accelerating with an acceleration $a = |\mathbf{a}|$ which represents a non-inertial (accelerating) reference frame N^a. A light ray is emitted from a point D on the elevator's wall (at equal distances h from the floor and the ceiling) and propagates in a direction perpendicular to the elevator's acceleration toward a point B on the opposite wall (also situated at equal distances h from the floor and the ceiling; the distance between D and B is h too). Due to the accelerated motion of the elevator the ray bends (for an observer in the elevator) and arrives not at the middle point B but at a point B' displaced at a distance δ from B toward the floor. This is the original thought experiment considered by Einstein. This is also an experiment that allows an observer in N^a (in the elevator) to determine from within N^a that it is not an inertial frame. In order to calculate the anisotropic velocity of light in N^a, in addition to the horizontal light ray propagating from point D toward point B, let us consider two other light rays propagating vertically along the line of acceleration which are emitted from two points on the wall where point B is situated. The two light rays are emitted simultaneously with the horizontal ray in N^a from two points A and C separated by a distance $2h$, one from point C (at the elevators floor) toward B (in the direction of the acceleration), and the other from point A (at the ceiling) in the opposite direction (also toward B). They will not meet at the middle point B because during the time $t = h/c$, the light rays travel toward B, N^a will move at a distance $\delta = at^2/2$ as measured in the co-moving (instantaneous) inertial reference frame. In N^a the three rays will meet at a point B' which is displaced from the middle point B (in the direction opposite to the acceleration) by the same distance determined in the instantaneous inertial frame:

$$\delta = \frac{1}{2}at^2 = \frac{ah^2}{2c^2}.$$

Here $t = h/c$ is also the time it takes for the light rays to meet at B (when the elevator is moving with constant velocity) which equals the time t in N^a for which the light rays simultaneously reach point B' after having traveled the distances $DB' \approx h$, $AB' = h + \delta$, and $CB' = h - \delta$. This shows that in N^a the speed of the upward light ray from point C to point B' (in the direction of the acceleration) is smaller than that in the opposite direction (and than c) and its average value is

$$c_{\uparrow}^{a} = \frac{x - \delta}{t} = c\left(1 - \frac{ah}{2c^2}\right). \tag{1}$$

The average velocity of the downward light ray from A to B' (in the direction opposite to the acceleration) is greater than c:

$$c_{\downarrow}^{a} = \frac{x + \delta}{t} = c\left(1 + \frac{ah}{2c^2}\right). \tag{2}$$

Therefore an observer in N^a will conclude that the three light rays arrive at B' (not at B) due to the anisotropy in the propagation of light in the elevator which in turn is caused by its accelerated motion.

As seen from (1) and (2) the average anisotropic velocity of light in N^a involves accelerations and distances for which $ah/2c^2 < 1$. This restriction is always satisfied since it is weaker than the one imposed by the principle of equivalence which requires that only small regions in a gravitational field where the field is uniform are considered [4].

If we now consider an elevator (i.e. a non-inertial reference frame N^g) at rest on the Earth's surface we can obtain the same average velocities (1) and (2) of the light rays emitted from point C (toward B) and from A (toward B), respectively. The elevator will appear accelerating upward (with an acceleration $g = |\mathbf{g}|$) with respect to an inertial (falling) reference frame I. During the time the light rays emitted from the points A, C, and D travel toward B the elevator will appear to have moved with respect to I at a distance $\delta = gt^2/2 = gh^2/2c^2$ and for this reason the three light rays will meet not at B but at B' situated bellow B at a distance δ. Therefore an observer in N^g (in the elevator) also finds that the propagation of light is anisotropic in the elevator. The average velocity of the light ray traveling from C to B' against $\mathbf{g}$ is

$$c_{\uparrow}^{g} = \frac{x - \delta}{t} = c\left(1 - \frac{gh}{2c^2}\right). \tag{3}$$

The average velocity of the light ray propagating downward from A to B' is greater than c [5]:

$$c_{\downarrow}^{g} = \frac{x + \delta}{t} = c\left(1 + \frac{gh}{2c^2}\right). \tag{4}$$

The velocities (3) and (4) can be also obtained from the expression for the velocity of light in a gravitational field derived by Einstein [6] in 1911 (see [8] and [9]). One can get the average velocities (3) and (4) directly from (1) and (2) by using the equivalence principle and substituting $a = g$ in (1) and (2).

As seen from (3) and (4) the anisotropic velocity of light in a gravitational field is a corollary of general relativity (more precisely a corollary of the equivalence principle). That is why an experiment for testing that anisotropy will not be a test of a new effect but another test of general relativity.

The purpose of this paper is to propose an experiment to test the anisotropic velocity of light in a gravitational field. The experiment to be described bellow is based on another experiment which was proposed by Stolakis [10] in 1986 with the intention to measure the one-way velocity of light. It turned out that this could not be done but it was pointed out that the experiment he proposed might be used for testing a possible anisotropy in spacetime [11]. The experiment can be described in the following way. Consider again the Einstein elevator in the Earth's gravitational field. At point B a light beam is split into two beams 1 and 2 which propagate vertically (with respect to the Earth's surface). Ray 1 travels the distance h upward from B to A in a medium of index of refraction n; at A it is reflected by a mirror and its return path toward B is in vacuum ($n = 1$). Ray 2 also travels the same distance h in a medium of index of refraction n but downward from B

to C; its return path after being reflected by a mirror at C is in vacuum too. Upon their arrival at point B rays 1 and 2 interfere. If the velocity of light is anisotropic the interference pattern produced by vertically propagating rays will differ from the interference pattern of two horizontally traveling rays.

Taking into account (3) and (4) the upward average velocity of ray 1 from B to A is

$$c_n^{\uparrow} = \frac{c_{\uparrow}^{g}}{n} = \frac{c}{n}\left(1 - \frac{gh}{2c^2}\right).$$

while the downward average velocity of ray 2 from B to C is

$$c_n^{\downarrow} = \frac{c_{\downarrow}^{g}}{n} = \frac{c}{n}\left(1 + \frac{gh}{2c^2}\right)$$

Then the time for which ray 1 goes to A and returns to B is

$$t^1 = \frac{h}{c_n^{\uparrow}} + \frac{h}{c_{\downarrow}^{g}} \approx \frac{hn}{c}\left(1 + \frac{gh}{2c^2}\right) + \frac{h}{c}\left(1 - \frac{gh}{2c^2}\right).$$

The time for which ray 2 goes to C and returns to B is

$$t^2 = \frac{h}{c_n^{\downarrow}} + \frac{h}{c_{\uparrow}^{g}} \approx \frac{hn}{c}\left(1 - \frac{gh}{2c^2}\right) + \frac{h}{c}\left(1 + \frac{gh}{2c^2}\right).$$

The difference between the two time intervals t^1 and t^2 is

$$\Delta t = t^1 - t^2 = \frac{gh^2}{c^3}\left(n - 1\right). \tag{5}$$

If the return paths of rays 1 and 2 are in a medium of index of refraction n' then obviously (5) becomes

$$\Delta t = \frac{gh^2}{c^3}\left(n - n'\right).$$

Performed as described here the experiment cannot detect a change in the interference pattern of two light rays one of which is delayed by Δt that is $\sim 10^{-23}s$ for $h = 10m$. This tiny delay is equivalent to a shift of the wavelength of one of rays with respect to the other's wavelength by $10^{-5}nm$ which cannot produce observable interference pattern. If, however, the two rays are made to pass multiple times through the medium toward points A and C, respectively and through the vacuum back to point B, the delay Δt accumulates and the effect may become detectable.

This experiment may turn out to be of crucial importance for the interpretation of general relativity. If the result is negative (i.e. $\Delta t = 0$), which is highly unlikely, this would mean that the average velocity of light in a gravitational field is c (only in this case $\Delta t = 0$) and therefore general relativity should be re-examined since it predicts the anisotropic velocity of light. If the result is positive, proving the anisotropy in the propagation of light in a gravitational field, general relativity will be once again confirmed. A positive outcome of the experiment, however, despite confirming general relativity *itself* may also affect its standard interpretation since the average anisotropic velocity of light implies that the local light velocity is not c at the observation point while according to the standard curved-spacetime interpretation of general relativity the local velocity of light is always c [6], [12]. Taking into account (4) and the fact that the initial velocity of light is c the local velocity of the A-photon (emitted from A) at B is

$$c_{\downarrow}^{A} = c\left(1 + \frac{gh}{c^2}\right). \tag{6}$$

Therefore the local velocity of the A-photon at B is not c. As seen from (6) the A-photon's velocity depends on the difference of the gravitational potential gh of the source and observation point; the A-photon's velocity is equal to c only at the source point A (where $h = 0$). Another indication that the local velocity of light depends on the difference of the gravitational potential of the light source and the observation point comes from the gravitational redshift [13].

It cannot be argued that the velocity of the A-photon at B is greater than c as given by (6) but this is *relative* to an observer at the source point A while an observer at B would measure that the local velocity of the A-photon at B is c. Such an argument leads to a contradiction with the fact that the greater than c average velocity of the A-photon, as seen from (4), is caused by the increasing velocity of the A-photon which is *falling* in the Earth's gravitational field (the A-photon is represented by a geodesic worldline which means that for a non-inertial observer at rest in the Earth's gravitational field the A-photon is falling). If we assume that for the B-observer the local velocity of the A-photon is c at B it follows that for the B-observer the initial velocity of the A-photon at A should be given by (6) - only in this case the A-photon's average velocity (which the proposed experiment intends to test) is equal to the average velocity given by (4). It is obvious, however, that this would mean that from B-observer's viewpoint the velocity of the A-photon on its way from A to B would be decreasing from $c\left(1 + gh/c^2\right)$ at A to c at B while, in fact, the A-photon's velocity is increasing due to its fall in the Earth's gravitational field (which is true for the B-observer as well). Therefore the B-observer also concludes that the local velocity of the A-photon at B is not c and is given by (6).

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[3] A. Einstein, L. Infeld, *The Evolution of Physics*, (Simon & Schuster, New York, 1966).

[4] The equivalence principle can be applied only to regions of dimension h in a gravitational field which are small enough (such that $gh/2c^2 \ll 1$) in order to ensure that the field is uniform there.

[5] The average velocities of light (3) and (4) can be directly obtained by noting that the propagation of light is affected by the Earth's gravity. At B' the instantaneous velocity of the ray emitted at C will be slowed down by the Earth's gravity: $c_{\uparrow}^{B'} = c - gt \approx c(1 - gh/c^2)$. As the initial velocity of the ray is c its average velocity from C to B' is $c_{\uparrow}^{g} = c(1 - gh/2c^2)$. Analogously the ray emitted at A will arrive at B' with the instantaneous velocity $c_{\downarrow}^{B'} = c + gt \approx c(1 + gh/c^2)$ and the ray's average velocity from A to B' will be $c_{\downarrow}^{g} = c(1 + gh/2c^2)$.

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